Journal of Economic Behavior & Organization
Vol. 56 (2005) 273–295

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

On the pareto-optimality of futures contracts over Islamic forward contracts: implications for the emerging Muslim economies

M. Shahid Ebrahim[a,*], Shafiqur Rahman[b]

[a] *Nottingham University Business School (Jubilee Campus), Wollaton Road, Nottingham NG8 1BB, UK*
[b] *School of Business Administration, Portland State University, USA*

Received 29 April 2002; received in revised form 30 April 2003; accepted 18 September 2003
Available online 30 July 2004

Abstract

A general equilibrium approach is used to demonstrate that: (i) futures contracting (on Islamically permissible commodities) is *pareto-optimal* over the Islamic forward contract of *Bai' Salam*; and (ii) both forms of contracting constitute a *quasi-equity* claim instead of debt (*dayn*) as construed by the majority of Islamic jurists. These results are of import as they: (i) remove a major hurdle against futures contracting by the Islamic jurists thereby enabling the renovation of the financial intermediation system of emerging Muslim economies; and (ii) demonstrate that the arbitrage principle needs to be re-examined under non-linear asset pricing.

JEL classification: D58; D74; D91; G12; G24; G32; P43

Keywords: Security design; Islamic banking

1. Introduction

This paper examines how emerging Muslim countries can benefit from developing their financial markets by incorporating futures contracts. The rationale behind this stems from

* Corresponding author. Tel.: +44 115 84 67654; fax: +44 115 84 66667.
E-mail address: m.shahid.ebrahim@nottingham.ac.uk (M.S. Ebrahim).

0167-2681/$ – see front matter
doi:10.1016/j.jebo.2003.09.007

Demetriades et al. (2000), who propagate the view that a good financial intermediation system can contribute significantly to the growth of a nation. We investigate the pareto-optimality of a "synthetic" futures contract over Islamic forward contract known as *Bai' Salam*.[1] The synthetic futures contract is a package that is financially engineered by combining futures contract on Islamically permissible commodities and Islamic cost-plus sale contract (*Bai' Murabahah*). We demonstrate that such a financially engineered package meets all the requirements of Islamic jurisprudence and dominates Islamic forward contract on efficiency and welfare issues. This result is contrary to the intuition that under competitive markets, arbitrage-free first-order conditions lead to pareto-neutrality of both contracts.

Islam, an Abrahamic religion, endorses free markets, discourages price controls and forbids financial contracts based on *riba*, *gharar* and *maysir* as explicated below (Islahi, 1988):

(i) *Riba* literally means an increase, addition, expansion, or growth, or the "premium" that must be paid by the borrower to the lender with the principal as a condition for the loan or for an extension of its maturity. However, *riba* has some very broad connotations, as expounded by the well-known Islamic jurist Ibn Qayyim Al-Jawziyya (1973) to imply: (a) unfairly trading in any form, manipulating the market or engaging a market participant to trade under duress (*riba-al-fadl*); and (b) interest-based debt contracts (*riba-an-nasi'ah*) (Fazlur-Rahman, 1969; Saeed, 1996). Ibn Qayyim rationalizes the prohibition of interest transactions in an era where the bulk of society lived in bare subsistence and were prone to exploitation by lenders. Nonetheless, the majority of contemporary Islamic scholars (termed as the Neo-Revivalists by Saeed) still rationalize its prohibition in Islam based on the social impact of bankruptcies and loan defaults emanating from excessive debt obligations.[2,3]

(ii) *Gharar* in a financial contract entails deception.

(iii) *Maysir*: Promoting *gharar* pre-empts *maysir*, which is gambling (*qimar*) (Ibn Taymiya, 1951, n.d.).[4]

---

[1] Please refer to the Glossary in Appendix for further exposition of Arabic terms associated with Islamic finance.

[2] It should be noted that the scriptures of other Abrahamic religions (before *Islam*) also proscribe interest. For example, in a letter, Pope Urban III (1185–1187) cited the words of Christ, "lend freely, hoping nothing thereby" (Luke 6:35) (Hastings, 1922). In Judaism, there are three Biblical passages (Exodus 22:24; Leviticus 25:36–37; Deuteronomy 23:20–21) that forbid taking interest from "brothers," but permit it when the borrower is a Gentile (non-Jew). In Leviticus, "increase" is the rendering of the Hebrew "marbit" or "tarbit" that denotes gain on creditor's side. Lending on interest is considered by Ezekiel (18:13, 17) among the worst sins. Also, in Psalm 15, among the attributes of the righteous man is the fact that he does not lend on usury (Anon., 1901, p. 338). See Keen (1997) for an excellent discourse on the shift in the attitude in the West from complete prohibition of interest to its acceptance.

[3] The evolution of the Islamic banking industry is attributed to the literal view of the Neo-Revivalists that all forms of trading money (or monetary equivalents) for more money over time constitutes *riba-an-nasi'ah*. In contrast to the Neo-Revivalists there is a minority of scholars (termed as Modernists by Saeed), who believe that *riba-an-nasi'ah* proscribed in the Muslim holy book (*Qur'an*) is the exploitative (i.e., the usurious) one.

[4] Incidentally, the elements of *gharar* and *maysir* have the capacity to impair the reputation of the financial services industry. This is precisely the reason why regulations in the developed economies encourage fair credit reporting laws and full disclosure laws and restrict insider trading.

Since exchanging money for more money (or monetary equivalents) at a fixed predetermined rate is considered *ribawi* (interest bearing) in Islam, financial instruments such as the debt (*Bai*' *Murabahah*) or equity (*Musharakah*) facilities are carefully structured so that the exchange involves goods for money (or partnership shares for money) over time.[5] Furthermore, the financier should also be subject to the risk of investment, in accordance with the Prophetic tradition (*hadith*) that entitlement of return from an asset vests on one bearing risk of it (*al-kharaj bi al-daman*) (Saeed). The purpose of financing is to facilitate trade or business and not to avoid the religious injunction.

The recent trend towards Islamic banking reinforces the above religious norms. However, in many Muslim countries financial intermediation is in the rudimentary stage of a banking system in need of augmentation from capital markets and especially futures markets, as their economies are predominately natural resource oriented.[6] Mainstream financial economists recognize the fact that futures trading reallocates risk, reduces price volatility, offers liquidity, leads to price discovery, and enhances social welfare (Francis, 2000; Goss, 2000). However, futures trading has been plagued by many misapprehensions by contemporary Islamic scholars. These scholars proscribe futures trading based on religious injunctions on: (i) gambling (*qimar*) as it is deemed to be a speculative activity; (ii) short-sales of goods/assets not owned or possessed by seller; (iii) the delay of both goods/assets and price in a transaction; and (iv) offsetting of one futures position with another as it is deemed to be the sale of one form of debt against another (*Bai*' *al dayn bi al dayn*). Furthermore, these contemporary scholars are in favor of an Islamic forward contract called *Bai*' *Salam* that was prevalent in the medieval period (Udovitch, 1975).[7] This instrument differs from the conventional futures contract in that full cash payment must be made at the initiation of the contract and that the underlying generic asset be normally available and traded in the markets at the maturity of the contract (Bacha, 1999; Zaman, 1991). Despite the constraints imposed by full cash payment in *Bai*' *Salam* at the origination of the contract, contemporary Islamic economists consider it to be a panacea for problems plaguing the contemporary Muslim countries, ranging from financing agricultural ventures to deficit financing by their governments (Khan, 1997; El-Gari, 1997).

In a path-breaking paper, Kamali (1996) refutes the allegations against conventional futures trading since: (i) it serves an economic purpose of reducing systematic price risk and should not be deemed as gambling (*qimar*) prohibited in the *Qur*'*an*; (ii) the short-sales restricted in the tradition (*Sunnah*) of Prophet Muhammad are on unique goods/assets and

[5] It should be noted that the Arabic word '*Bai*' implies sale. *Bai*' *Murabahah* represents the cost-plus (*Murabahah*) sale while *Bai*' *Salam* described further connotes the Islamic forward (*Salam)* sale.

[6] The current state of Islamic banking has been severely criticized by Aggarwal and Yousef (2000) and Errico and Farahbaksh (1998), among others. One of the problems pointed out is the excessive use of the *Murabahah* structured in a way to resemble a financial facility bearing a fixed (*ribawi*) rate of return without being subject to risk. These missteps and miscues of Islamic banks could have been avoided had they resorted to a carefully drawn out planning (*Ijtihadi*) process involving academics, practitioners, and the Islamic scholars as espoused by Al-Alwani (1997) and Saeed (1996).

[7] This is originally narrated in a tradition of Prophet Muhammad from *Sahih* Muslim roughly 1424 years ago by Ibn Abbas who reported that "when God's Apostle (PBUH) migrated to Medina people were paying 1 and 2 years in advance for fruits, so he said: Those who pay in advance for anything must do so for a specified weight and time" (see Siddiqui, 1986; *Hadith* # 3906).

not on generic (fungible) goods/assets; (iii) possession (*qabd*) of goods/assets prior to sale is not a prerequisite for avoiding deception (*gharar*) as delivery is guaranteed by the futures clearing house; and (iv) the jurists proscription of delaying both goods/assets and price in a sale and the offsetting of futures position with another have no support in the *Qur'an* or the authentic traditions (*hadith*) of Prophet Muhammad. Kamali (1996) thus concludes that futures trading is Islamically permissible (*ibahah*) as long as it excludes contracts on non-permissible commodities and those deriving their substance from interest (*ribawi*) elements such as interest rate futures.[8]

The primary purpose of this paper is to demonstrate that a "synthetic" futures contract is pareto-optimal over the *Bai' Salam* contract, which is a forward contract on an Islamically permissible commodity with full (100 percent) margin deposit.[9] Intuitively speaking, *Bai' Salam* can be conceptualized as a linear combination of a conventional futures contract (with nominal margin deposit) along with a debt facility compatible in Islam (*Bai' Murabahah*). Payoff from a long (short) *Bai' Salam* can be replicated by buying (selling) a futures contract on an Islamically permissible commodity and investing (going short) in an Islamic debt instrument (*Bai' Murabahah*). The debt instrument will be for an amount equal to the revenue gained from the futures transaction net of margin deposit and can be construed as a cash payment in exchange for future delivery of the underlying commodity. This replication results in a synthetically created futures contract by combing an Islamic forward contract and an Islamic debt instrument.[10] In competitive markets, arbitrage-free first-order conditions require that both contractual packages be equally efficient. However, our unique result of pareto-efficiency of "synthetic" futures package over that of the Islamic forward contract is attributed to the fact that unconstrained optimization (of a linear sum of securities) is better than a constrained optimization (of a single equivalent security). This result is of import to mainstream economists who use the principle of arbitrage to price equivalent sets of securities (especially derivative securities) using linear valuation schemes as espoused in Varian (1987). However, this equivalence does not hold when the components comprising the financial packages themselves are non-linear. This issue needs to be re-examined as it impacts social welfare.

A secondary goal of this paper is to illustrate from our intermediate results that pre-selling the underlying asset in an Islamic forward or futures contract constitutes selling equity in the production process. *Although this creates an obligation for the seller to deliver*

[8] Kamali (2002) takes the above arguments further by challenging the orthodox views held by many Islamic scholars who prohibit many contemporary financial instruments that do not meet their rigid criteria. We are grateful to an anonymous referee for bringing this to our attention.

[9] The analysis of pareto-efficiency (that no person or group can be made better off without another being made worse off) is not alien to Islamic jurisprudence. It is cited under equity in Islamic Law (*Istihsan*) and has been deduced from the Prophetic injunction (*hasan hadith*) "No harm shall be inflicted or reciprocated in Islam," narrated in Ibn Majah, Sunan II, 784, *Hadith* # 2340 (see Kamali, 2000).

[10] In general, differences between futures and forward prices for short-term contracts with settlement dates less than 9 months tend to be very small. That is, the daily marking to market process appears to have little effect on the setting of futures and forward prices. Moreover, if the underlying asset's returns are not highly correlated with interest rate changes, then the marking to market effects are small even for longer-term futures. Only for longer-term futures contracts on interest-sensitive assets will the marking to market costs be significant. Because of this, it is a common practice in the literature to analyze futures contracts as if they were forwards. For details see Ritchken (1996).

the commodity in the grade and quantity negotiated and resembles a liability, it does not constitute a debt on the part of the seller as construed by Islamic jurists. This issue is important as one of the points raised by contemporary Islamic scholars against the offsetting transaction in futures contract (to close out the position) stems from the jurists' injunction of sale of debt by debt (*Bai' al dayn bi al dayn*). It is also regarded as a major impediment to the development of futures market in Muslim countries as cited by Vogel and Hayes (1998). If futures contracts do not constitute such an exchange of debt for debt, then there is no problem in trading them. Thus, our results augment those of Kamali (1996, 2002).

Even though the above result, identifying Islamic forwards and futures as quasi-equity instruments, is contrary to the prevailing view of the majority of contemporary Islamic scholars, it is still permissible according to the Islamic law (*Shari'ah*). This is because the *Qur'an* advises Muslims to revert to it and the practice of the Prophet (*Sunnah*) in case of differences with those in authority (implying religious or political authority) over them.[11] Furthermore, Prophet Muhammad explicitly demarcated the authority of religious scholars to that of spiritual matters and not on issues of technical nature (implying various scientific fields that can be interpreted as including modern financial economics).[12,13]

Our results therefore provide the impetus to the emerging Muslim countries to establish futures markets and benefit from the effects of financial depth. Currently, there are few countries that do so with contracts limited to few products. These are Indonesia (coffee and crude palm oil), Kazakhstan (wheat), Malaysia (crude palm oil, stock index and government debt) and Turkey (currency) (Bacha, 2002; Kamali, 2002; Peck, 2000).[14] In contrast, there is a limited amount of over the counter trading in many Muslim countries using *Bai' Salam*. The information on this is not publicly available except for the case of Iran, where it has

[11] This is cited in the following verse of the *Qur'an* (4:59): "O ye who believe! Obey God, His Apostle and those charged with authority among you. If ye differ in anything among yourselves, refer it to God and His Apostle, if ye do believe in God and the Last Day: That is best, and most suitable for final determination."

[12] This is cited in Chapter 986 of *Sahih* Muslim titled as "It is obligatory to follow the Prophet (PBUH) in all matters pertaining to Religion, but one is free to act on one's own opinion in matters pertaining to technical skill." "Rafi' b. Khadij reported that the Prophet (PBUH) saw the people grafting the trees when he migrated to Medina. He inquired: What are you doing? They replied: We are grafting them, whereupon he expressed his disapproval by saying: It may be good for you if you do not do that, so they abandoned this practice, (and the date-palms) began to yield less fruit. They made a mention of it (to the Prophet), whereupon he said: I am a human being, so when I command you about a thing pertaining to religion, do accept it, and when I command you about a thing out of my personal opinion, keep in mind that I am a human being. '*Ikrima* reported that he said something like this." (see Siddiqui, 1986; *Hadith* # 5831).

[13] A broadminded perspective is also espoused by the well known Islamic economist, M.N. Siddiqi, in the following quotations:

> "Islamic finance is open to any innovations that are in congruence with its fundamentals." (Siddiqi, 2002)

> "We should never lose sight of the reality that the divine part of modern Islamic finance, though crucial, is very small. The rest is man-made resulting from *Ijtihad* (efforts in understanding and applications)." (Siddiqi, 2001)

[14] The reason why some of the above Muslim countries allow futures trading on non-permissible assets such as debt and currency is because they either have a dual (Islamic + conventional) system (in case of Malaysia) or follow a purely conventional system (in case of Turkey).

constituted 5–5.9 percent of all financing by Islamic banks during the period 1995–1998 (Yasseri, 2000).

In the context of modern asset pricing theory, futures contracts are priced using the concept of: (a) systematic risk as in Capital Asset Pricing Model (CAPM)/Consumption Capital Asset Pricing Model (CCAPM) (Kolb, 1996; Breeden, 1980); (b) Hedging-Pressure Explanation (Hirshleifer, 1988); or (c) General Equilibrium Theory (Francis, 2000). We prefer to use the general equilibrium approach as the CAPM or the Hedging-Pressure theories would fail to distinguish the efficiencies of the alternative financing packages considered herein under an evenly distributed demand function. Furthermore, our approach has a strong following in the academic and policy communities.[15] Our model incorporates the quantity (yield) risk in conjunction with the price risk consistent with the views of Hirshleifer (1975). We evaluate the welfare of agents (hedgers) in the economy in the *Salam* sale contract after pricing it optimally and contrast it with a "synthetic" futures contract.[16] This estimation is performed using both theoretical assertions as well as with a numerical example emulating Mehra and Prescott (1985) and Kocherlakota (1996) who have advocated its use in asset pricing. The paper is organized as follows: the modeling of *Bai' Salam*, synthetic futures and their respective solutions are explicated in Sections 2–4 and then further elaborated with a numerical example in Section 5. Finally, Section 6 provides some concluding remarks.

2. Modeling the Islamic forward (*Bai' Salam*) contract

For simplicity and mathematical tractability, we assume a two-period economy with two types of production technologies. The first production process is for the commodity and the second is for a final good that uses the commodity as input. There are two types of agents, the producer and user of a basic commodity. The risks that these agents face are the production yield ($\tilde{m}$) and the final price ($\tilde{x}$) of a commodity as illustrated in Fig. 1.[17] These two types of uncertainties are exogenously represented by two correlated stochastic variables $\tilde{m}$ and $\tilde{x}$. Depending on the sign of this correlation (ranging from negative to zero to positive), the commodity can be construed to be an inferior, intermediate or a normal good (Siegel and Siegel, 1994). Initially there is an Islamic forward market for the commodity. There is also a spot market for commodity once the uncertainty is resolved at $t = 1$. This spot market is automatically cleared as all the remaining commodity available is bought by the user. All of the agents maximize their expected utility of consumption by choosing their optimal position on *Bai' Salam* contract. To find equilibrium for this economy, the Islamic forward market has to clear as illustrated below.

[15] Whalley (1988) summarizes the experience of economists with different general equilibrium models and discusses how they can be used in the policy process.

[16] Due to the difference in timing of payments in the two contracts, one has to introduce an Islamic facility of cost plus: *Bai' Murabahah*.

[17] The producer [user] is subjected to risk when the price of the basic commodity is low [high] at $t = 1$. The agents can hedge their respective risk by entering into a *Bai' Salam* contract.

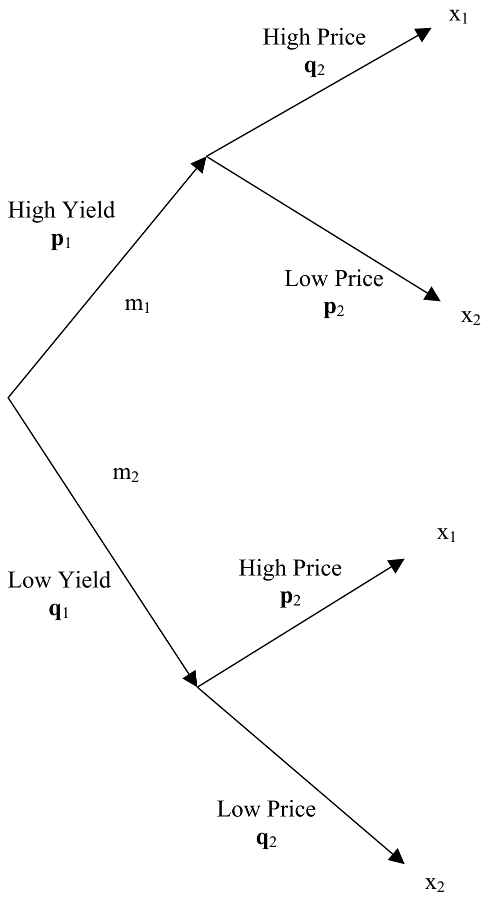

Fig. 1. Interrelationship between yield and price.

2.1. Modeling the objective of commodity producer

The goal of commodity producer is to optimize the expected utility of consumption:

$$\max E_0\{U(c_0) + \beta U(\tilde{c}_1)\} \quad (\text{in } c_0, c_1, s, f_{\mathrm{S}}),$$

subject to the temporal wealth constraints:

$$c_0 = w_0 + s f_{\mathrm{S}}, \tag{1}$$

$$\tilde{c}_1 = w_1 + \tilde{x}(\tilde{m} - s), \tag{2}$$

where $E_0\{\cdot\}$ is the expectation operator at time $t = 0$, $U(\cdot)$ a differentiable and quasi-concave utility function, c_0 the consumption of commodity producer at $t = 0$, $\tilde{c}_1$ the consumption of commodity producer at $t = 1$, w_0 the endowment at $t = 0$, w_1 the endowment at $t = 1$, β the discount factor, $\tilde{m}$ the stochastic yield of the product at $t = 1$, s the amount of produce sold via the *Bai' Salam* contract, and f_S the unit price of the *Bai' Salam* contract.

The budget constraint at time $t = 0$ (Eq. (1)) illustrates consumption utilizing the initial endowment (w_0) and pre-selling s units of output at f_S. The budget constraint at $t = 1$ (Eq. (2)) incorporates consumption from future endowment (w_1) in addition to payoffs from the residual ($\tilde{m} - s$) units of output at the prevailing market price ($\tilde{x}$).

The Lagrangian L can be written as:

$$L = E_0\{[U(c_0) + \beta U(\tilde{c}_1)] + \lambda_0[w_0 + sf_S - c_0] + \lambda_1\beta[w_1 + \tilde{x}(\tilde{m} - s) - \tilde{c}_1]\}.$$

The Euler equation (first-order necessary condition or FONC) is given by:

(i) At the margin, the producer will sell fractional shares of the output that yield net benefits at least equal to zero. This yields optimal price of *Bai' Salam* (f_S) given as follows:

$$f_S \geq \beta E_0 \left\{ \left[\frac{U'(\tilde{c}_1)}{U'(c_0)} \right] [\tilde{x}] \right\}. \tag{3}$$

The above equation represents the *supply function* of the quantity of output pre-sold (s) at a unit price of f_S. It is equivalent to a two-period Lucas (1978) asset pricing equation. It should be noted that the output pre-sold using this contract characterizes a liability in the form of s units of production. *Therefore, it constitutes pre-selling equity in part of the production process and not debt as construed by the Islamic scholars*.

Thus, maximization of producer's objective requires that the following be satisfied:

(a) The deterministic budget constraint (at $t = 0$), as depicted in Eq. (1), and the stochastic budget constraint (for each state of the economy at $t = 1$), as shown by Eq. (2).
(b) The simplified FONC (Euler equation) [Eq. (3)].
(c) The second-order conditions for a maximum be satisfied. We do not attempt this as Chiang (1984) demonstrates that maximization of a differentiable quasi-concave objective function (such as a power utility) with linear constraints gives a negative definite bordered Hessian matrix.

2.2. *Modeling the objective of commodity user*

Similar to the previous case, the goal of the commodity user is to maximize the expected utility of consumption:

$$\max E_0\{V(c'_0) + \beta' V(\tilde{c}'_1)\} \quad (\text{in } c'_0, c_1, s', f_S),$$

subject to the temporal wealth constraints:

$$c'_0 = w'_0 - s' f_S, \tag{4}$$

$$\tilde{c}'_1 = w'_1 + (\tilde{m} - s')(d - \tilde{x}) + s'd, \tag{5}$$

where $V(\cdot)$ denotes the differentiable, quasi-concave utility function of the user, d the pre-negotiated unit-selling price of the finished goods, and the remaining notations with prime have the same meaning as those in the case of the producer.[18]

The budget constraint at time $t = 0$ (Eq. (4)) denotes consumption utilizing the residual of initial endowment (w'_0) after pre-payment of s' units of output at f_S. The budget constraint at $t = 1$ (Eq. (5)) depicts consumption from future endowment (w'_1) along with payoffs from: (i) pre-negotiated s' units of output at price d; and (ii) residual $(\tilde{m} - s')$ units of output at a stochastic profit margin $(d - \tilde{x})$.

The Lagrangian L' can be written as:

$$\begin{aligned} L' = {} & E_0\{[V(c'_0) + \beta' V(\tilde{c}'_1))] + \lambda_0[w'_0 - s' f_S - c'_0] \\ & + \lambda_1 \beta'[w'_1 + (\tilde{m} - s')(d - \tilde{x}) + s'd - \tilde{c}'_1]\}. \end{aligned}$$

The Euler equation (FONC) is given by the following:

(i) At the margin, the commodity user will purchase fractional shares of the output, which yield net benefits at most equal to zero. This again yields optimal price of *Bai' Salam* (f_S) given as follows:

$$f_S \leq \beta' E_0 \left\{ \left[\frac{V'(\tilde{c}'_1)}{V'(c'_0)} \right] [\tilde{x}] \right\}. \tag{6}$$

The above equation constitutes the *demand function* for s' units of product pre-negotiated at a unit price f_S.

Thus, maximization of the commodity user's objective requires that:

(a) The deterministic budget constraints in both periods represented by Eqs. (4) and (5) be satisfied.
(b) The simplified FONC (Euler Equation) [Eq. (6)] be satisfied.
(c) The second-order condition for a maximum be satisfied. Here, again we do not verify this based on the result of Chiang for a differentiable quasi-concave function such as a power utility.

3. Modeling the synthetic futures contract

We now model the alternative to a *Bai' Salam* contract, a synthetically created futures contract. The main difference between *Bai' Salam* and the futures contract is in the timing of payment. Although the futures contract is pre-negotiated at $t = 0$, it is consummated at $t = 1$ when the commodity is delivered and payment for it is made. Thus, in the case of futures contract, the producer of the basic commodity would finance his enterprise utilizing an Islamically permissible debt facility (*Bai' Murabahah*) and redeem it at time $t = 1$ when the output is sold on the spot and futures markets at a unit price $\tilde{x}$ and f_C, respectively.

[18] One unit of finished goods uses one unit of basic commodity as input. This is not a crucial assumption of the analysis.

3.1. Modeling the objective of commodity producer

The goal of the agent is to optimize the expected utility of consumption:

$$\max E_0\{U(c_0) + \beta U(\tilde{c}_1)\} \quad (\text{in } c_0, c_1, s, Q, r, f_C),$$

subject to the temporal wealth constraints:

$$c_0 = w_0 + Q, \tag{7}$$

$$\tilde{c}_1 = w_1 + \tilde{x}(\tilde{m} - s) + sf_C - Q(1 + r), \tag{8}$$

where f_C denotes the conventional futures contract, Q and r the pricing parameters for *Bai' Murabahah* contract and the remaining notations have the same meaning as in Section 2.1.

The budget constraint at time $t = 0$ (Eq. (7)) illustrates consumption materializing from the initial endowment (w_0) and utility of asset purchased using the *Bai' Murabahah* facility (Q). The budget constraint at $t = 1$ (Eq. (8)) involves consumption from future endowment (w_1) along with net-payoffs from: (i) pre-negotiated futures contract for s units of output at price f_C; (ii) residual units of output ($\tilde{m} - s$) at the prevailing market price ($\tilde{x}$); and (iii) *Murabahah* repayment of $Q(1 + r)$.

The Lagrangian L here can be written as:

$$L = E_0\{[U(c_0) + \beta U(\tilde{c}_1)] + \lambda_0[w_0 + Q - c_0] + \lambda_1\beta[w_1 + \tilde{x}(\tilde{m} - s) + sf_C - Q(1 + r) - \tilde{c}_1]\}.$$

The Euler equation, i.e., first-order necessary conditions are given by the following:

(i) At the margin, the intertemporal marginal rate of substitution (MRS) at most equals the discounted value of inputs of the production process financed by the *Bai' Murabahah* facility:

$$\beta E_0\left[\frac{U'(\tilde{c}_1)}{U'(c_0)}\right] \leq \frac{1}{(1 + r)}. \tag{9}$$

The above equation denotes the *demand function* for the credit facility.

(ii) At the margin, the agent will sell forward fractional shares of the output that yield net benefits at least equal to zero. This yields optimal price of conventional futures (f_C) given as follows:

$$f_C \geq E_0\left\{\frac{U'(\tilde{c}_1)\tilde{x}}{U'(\tilde{c}_1)}\right\}. \tag{10}$$

The above equation represents the *supply function* of s units of output pre-sold at a price f_C. Here again, this is akin to pre-selling part of equity of the production process. *It constitutes a liability on the part of the seller. However, contrary to the Islamic Jurists' claim, it is not a debt contract.*

Thus, maximization of investor's objective requires the following:

(a) The budget constraints (at $t = 0, 1$), as depicted in Eqs. (7) and (8), be satisfied.
(b) The simplified Euler equations (FONCs) [Eqs. (9) and (10)] be satisfied.

3.2. Modeling the objective of commodity user

Similar to the previous case, the goal of the commodity user is to maximize utility of expected consumption:

$$\max E_0\{V(c_0') + \beta' V(\tilde{c}_1')\} \quad (\text{in } c_0', c_1', s', Q', r, f_C),$$

subject to the temporal wealth constraints:

$$c_0' = w_0' - Q', \tag{11}$$

$$\tilde{c}_1' = w_1' + (\tilde{m} - s')(d - \tilde{x}) + s'(d - f_C) + Q'(1 + r), \tag{12}$$

where the notations have the same meaning as in Sections 2.2 and 3.1.

The budget constraint at time $t = 0$ (Eq. (11)) denotes consumption stemming from the initial endowment (w_0') after financing of asset purchased (by Agent 1) using the *Bai' Murabahah* facility (Q). The budget constraint at $t = 1$ (Eq. (12)) involves consumption from future endowment (w_1') along with net-payoffs from: (i) pre-negotiated futures contract for s' units of output at the profit margin of $(d - f_C)$; (ii) residual units of output $(\tilde{m} - s')$ at the profit margin $(d - \tilde{x})$; and (iii) *Murabahah* repayment of $Q(1 + r)$.

The Lagrangian L' can be written as:

$$\begin{aligned} L' = {} & E_0\{[V(c_0') + \beta' V(\tilde{c}_1'))] + \lambda_0[w_0' - Q' - c_0'] \\ & + \lambda_1 \beta'[w_1' + (\tilde{m} - s')(d - \tilde{x}) + s'(d - f_C) + Q'(1 + r) - \tilde{c}_1']\}. \end{aligned}$$

The Euler equations (FONCs) are given by the following:

(i) At the margin, the MRS at least equals present value of asset financed using *Bai' Murabahah* facility:

$$\beta' E_0 \left[\frac{V'(\tilde{c}_1')}{V'(c_0')} \right] \geq \frac{1}{(1 + r)}. \tag{13}$$

The above equation denotes the *supply function* for the credit facility.

(ii) At the margin, the commodity user will purchase fractional shares of the output that yield net benefits at most equal to zero. This again yields optimal price of conventional futures (f_C) given as follows:

$$f_C \leq E_0 \left\{ \frac{V'(\tilde{c}_1')\tilde{x}}{V'(\tilde{c}_1')} \right\}. \tag{14}$$

Here, again the above equation represents the *demand function* for s' units of output pre-negotiated at a price f_C.

Thus, maximization of the commodity user's objective requires that:

(a) The deterministic budget constraints in both periods represented by Eqs. (11) and (12) be satisfied.
(b) The simplified Euler equations, FONCs [Eqs. (13) and (14)] be satisfied.

4. Model solutions

Assuming competitive markets, a *unique interior* solution is feasible under both models discussed in Sections 2 and 3. We first examine the necessary market clearing conditions before evaluating the key pricing conditions under both systems. Finally, we contrast the two and derive our main result of pareto-optimality.

4.1. Necessary market clearing conditions

The market clearing conditions are as follows:

(i) For the *Bai' Salam*/futures markets to be in equilibrium, the fractional shares sold must equal that purchased ($s = s'$). Furthermore, Islamic principle does not allow selling negative quantity of output by the producer ($s = s' < 0$) or selling in excess of the minimum yield, $\min[\tilde{m}]$ (Kamali, 1996, 2002). This results in the following constraint:

$$\Rightarrow \min(\tilde{m}) \geq s = s' > 0. \tag{15}$$

(ii) For the Islamic credit market to be in equilibrium, price of intermediate goods purchased by producer = funds expended by user:

$$\Rightarrow Q = Q'. \tag{16}$$

The models are solved below for the two distinct types of hedgers.

4.2. Key results

The market clearing conditions are as follows:

Proposition I. *A general equilibrium with Bai' Salam necessitates satisfaction of the following derivative pricing condition*:

$$f_S = \beta E_0 \left\{ \left[\frac{U'(\tilde{c}_1)}{U'(c_0)} \right] [\tilde{x}] \right\} = \beta' E_0 \left\{ \left[\frac{V'(\tilde{c}'_1)}{V'(c'_0)} \right] [\tilde{x}] \right\}. \tag{17}$$

Proof. This follows from equating the *supply* and *demand* functions of *Bai' Salam* given by Eqs. (3) and (6), respectively. □

Proposition II. *A general equilibrium with synthetic futures necessitates satisfying either of the two conditions given below*.

Credit pricing condition:

$$\frac{1}{(1+r)} = \beta E_0\left[\frac{U'(\tilde{c}_1)}{U'(c_0)}\right] = \beta' E_0\left[\frac{V'(\tilde{c}'_1)}{V'(c'_0)}\right]. \tag{18}$$

Derivative pricing condition:

$$f_{\mathrm{C}} = E_0\left\{\frac{U'(\tilde{c}_1)\tilde{x}}{U'(\tilde{c}_1)}\right\} = E_0\left\{\frac{V'(\tilde{c}'_1)\tilde{x}}{U'(\tilde{c}'_1)}\right\}. \tag{19}$$

Proof. This follows from equating the *supply* and *demand* functions of *Bai' Murabahah* [futures] given by Eqs. (13) and (9) [Eqs. (10) and (14)], respectively. □

Theorem. *A synthetic futures contract is pareto-optimal over Bai' Salam in a general equilibrium with risk averse agents.*[19]

Proof. An equilibrium with *Bai' Salam* is more restrictive than the one with synthetic futures as it requires 100 percent down payment at the onset of contracting (i.e., at t = 0). Since welfare of agents in a constrained optimization is lower than an unconstrained one, the result is that an equilibrium with *Bai' Salam* is *pareto-inferior* to that of synthetic futures. □

5. Numerical illustration

Since the models depicted in Section 4 involve the non-linear interaction between r, Q, s and $f_{\mathrm{S}}/f_{\mathrm{C}}$, their closed-form solutions are difficult to evaluate under agent risk aversion. The systems of equations encompassing the endogenous variables are numerically solved, using simulation methodology described below.

5.1. Model calibration

The various exogenous parameters adopted for the model being simulated are described below:

[19] It should be noted that under risk-neutrality both schemes are *pareto-neutral* (or *arbitrage-free*) as long as $\beta = \beta'$. The futures is priced at the well-known expectations hypothesis, while *Bai' Salam* is priced at a discounted value of it. The closed-form solution under risk-neutrality is given as follows:

$$f_{\mathrm{C}} = E_0(\tilde{x});\ f_{\mathrm{S}} = \beta E_0(\tilde{x}) = \frac{E_0(\tilde{x})}{1+r};\ s\in[0, \min(\tilde{m})];\ Q\in[0,\ f_{\mathrm{S}} \min(\tilde{m})] \text{ and } r = \frac{1-\beta}{\beta}.$$

(1) We select a discount factor ($\beta = \beta'$) of 0.99 and constant relative risk aversion (CRRA) utility function to represent the behavior of agents in the economy following the methodology of Mehra and Prescott.[20] The coefficient of risk aversion (α) is varied in a range [0.1, 5] in accordance with Kocherlakota.
(2) Aggregate endowments in periods zero and one are chosen as 2 and 0.4, respectively. The commodity user (also known as Agent 2) is granted endowments which are a multiple 'n' of those of the producer (also known as Agent 1) in both time periods.
(3) The stochastic output is assumed to follow a binomial distribution such that a high yield of $m_1 = 2$ and a low yield of $m_2 = 1$ occurs with probabilities 0.6 and 0.4, respectively. Furthermore, the price is negatively correlated with the yield as shown in Fig. 1. When the yield is high, then price is low (x_2) with probability $p_2 = 0.8$ and price is high (x_1) with probability $q_2 = 0.2$. Similarly, when the yield is low, then price is high (x_1) with probability p_2 and low (x_2) with q_2.[21]

5.2. Simulation methodology

The simulation is conducted by solving Eq. (18) for the *Bai' Salam* model and Eqs. (18) and (19) simultaneously for the synthetic futures model in the following procedure:[22]

(i) The first set of results given in Table 1 are evaluated assuming that the agents in the economy equally share half of the aggregate endowment in both periods and have the same level of risk aversion ($w_0 = w_0' = 1$, $w_1 = w_1' = 0.2$ and $\alpha = \alpha_1 = \alpha_2$).
(ii) The second group of results given in Table 2 assumes only the equality of risk aversion ($\alpha = \alpha_1 = \alpha_2$) and allows for the inequality of endowments.
(iii) The third set of results in Table 3 allows for the inequality of both endowments and level of risk aversion.
(iv) Finally, sensitivity analysis is conducted to evaluate the change in endogenous parameters due to a single change in each of the exogenous parameters. These results are not reported and are available on request from the authors.

5.3. Simulation results

Case I. Table 1 illustrates the results from both models, when both agents are equally wealthy and risk averse. Subcase A demonstrates an inverse relationship between s and f_S, whereas subcase B shows a monotonically increasing relationship of s and f_C. This

[20] A CRRA utility function for the producer is given by the following formula: $U(c_i) = c_i^{1-\alpha_1}/(1-\alpha_1)$, $\forall \alpha_1 \neq 1$; and $U(c_i) = \ln(c_i)$, for $\alpha_1 = 1$; where α_1 is his coefficient of risk aversion. A similar formula holds true for the user, whose utility is denoted by $V(c_i')$ and α_2 is his coefficient of risk aversion.

[21] This inherently assumes that the commodity under consideration has negative income elasticity and is construed as an 'inferior' good. This assumption is not crucial for our analysis as we are able to extrapolate our results for the contrasting case when the quantity (yield) risk and price risk are positively correlated to each other implying a positive income elasticity and thus a 'normal' good (Siegel and Siegel, 1994).

[22] The solution is obtained using *Mathematica Version 4* on a Windows platform.

Table 1
Simulation results (equal wealth and risk aversion)

α	s	f_S	$E(PV(x))$	Premium (percent)	SU(P)	SV(U)
(Subcase A) *Bai' Salam* model						
0.1	0.2778	0.9698	0.9680	0.1803	2.9339	1.7779
0.2	0.2784	0.9623	0.9588	0.3619	3.1897	2.0484
0.4	0.2791	0.9501	0.9433	0.7278	3.9807	2.8648
0.6	0.2793	0.9415	0.9313	1.0954	5.6045	4.5087
0.8	0.2790	0.9361	0.9226	1.4625	10.5482	9.4677
1	0.2783	0.9341	0.9173	1.8263	0.5698	−0.5003
2	0.2689	0.9700	0.9370	3.5213	−1.52767	−2.6181
3	0.2525	1.0799	1.0298	4.8620	−0.6011	−1.8491
4	0.2317	1.2669	1.1978	5.7697	−0.3153	−1.9331
5	0.2079	1.5438	1.4527	6.2750	−0.1831	−2.5937

α	s	f_C	r (percent)	Q	Premium (percent)	SU(P)	SV(U)
(Subcase B) Synthetic futures model: $E(x) = 0.988$							
0.1	0.7655	0.9897	2.0617	0.2628	0.1678	2.9340	1.7781
0.2	0.7582	0.9913	3.0361	0.2614	0.3376	3.1898	2.0487
0.4	0.7459	0.9947	4.7425	0.2588	0.6817	3.9809	2.8655
0.6	0.7368	0.9982	6.1111	0.2567	1.0296	5.6047	4.5099
0.8	0.7307	1.0016	7.1299	0.2550	1.3784	10.5484	9.4693
1	0.7276	1.0050	7.7935	0.2538	1.7255	0.5700	−0.4981
2	0.7490	1.0211	6.0333	0.2540	3.3532	−1.52771	−2.6100
3	0.8080	1.0341	−2.3550	0.2638	4.6697	−0.6017	−1.8223
4	0.8744	1.0436	−14.1910	0.2810	5.6317	−0.3167	−1.8499
5	0.9299	1.0501	−26.9185	0.3032	6.2899	−0.1851	−2.3399

The *Bai' Salam* and synthetic futures models are solved assuming the following exogenous parameters: (1) $w_0 = w'_0 = 1$, $w_1 = w'_1 = 0.2$, $\alpha = \alpha_1 = \alpha_2$, $d = 1.2$ and $\beta = 0.99$; (2) the yield and spot prices follow a binomial distribution such that both are negatively correlated as illustrated in Fig. 1. Furthermore, $m_1 = 2$, $m_2 = 1$, $p_1 = 0.6$, $q_1 = 0.4$, $x_1 = 1.1$, $x_2 = 0.9$, $p_2 = 0.8$, $q_2 = 0.2$. The endogenous parameters evaluated comprise the following: s (quantity of commodity bid for in either contract), f_S (*Bai' Salam* price), f_C (conventional futures' price), Q (*Bai' Murabahah* funding amount), and r (profit rate). Finally, SU(P) and SV(U) denote the sum of expected utilities of producer and user of commodity, respectively.

is attributed to the differences in the pricing in the two models. Under the *Bai' Salam* contract, lower quantity (s) is pre-sold at a lower price ($f_S < f_C$) as compared to conventional futures. However, the pricing mechanism in *Bai' Salam* (in general, with the exception of the subcase $\alpha = 5$) has a higher premium when compared to the expected present value of spot prices in contrast with futures. The *Bai' Murabahah* rate of return (supplementing the futures contract) increases at first with increasing risk aversion, but declines after $\alpha = 2$. It is negative for $\alpha \in [3, 5]$ due to our restriction that any excess funds left over from consumption have to be invested at even negative rates to smooth out consumption.[23] From

[23] This has some credence in the real world where fixed income securities may not perfectly hedge inflationary shocks resulting in negative returns.

an Islamic perspective negative returns indicate that the facility involves loss of capital. This is not *ribawi* as it satisfies the prerequisite stated earlier in Section 1 that the financier in *Bai*' *Murabahah* should be subject to risk. Nonetheless, comparing the sum of utilities of both agents, we conclude that the synthetic futures contract is pareto-optimal over the *Bai*'

Table 2
Simulation results (unequal wealth-equal risk aversion)

n	s	f_S	$E(PV(x))$	Premium (percent)	SU(P)	SV(U)
(Subcase A) *Bai*' *Salam* model: α = 0.4						
0.1	0.0000	–	0.9441	–	4.8080	1.5149
0.2	0.0148	0.9551	0.9429	1.2903	4.6632	1.8256
0.4	0.1051	0.9513	0.9422	0.9612	4.4280	2.2376
0.6	0.1768	0.9502	0.9424	0.8294	4.2456	2.5126
0.8	0.2335	0.9500	0.9428	0.7638	4.0999	2.7123
1	0.2791	0.9501	0.9433	0.7278	3.9807	2.8648
2	0.4154	0.9521	0.9457	0.6836	3.60736	3.291003
4	0.5217	0.9555	0.9488	0.7054	3.2885	3.6050
6	0.5658	0.9575	0.9506	0.7303	3.1451	3.7335
8	0.5899	0.9588	0.9517	0.7486	3.0635	3.8034
10	0.6049	0.9597	0.9525	0.7621	3.0108	3.8473

n	s	f_C	r (percent)	Q	Premium (percent)	SU(P)	SV(U)
(Subcase B) Synthetic futures model: α = 0.4 and $E(x)$ = 0.988							
0.1	1.0000	1.0028	4.7577	−0.0668	1.4960	4.8169	1.5341
0.2	1.0000	0.9992	4.8967	−0.0101	1.1316	4.6684	1.8339
0.4	1.0000	0.9961	4.9292	0.0837	0.8205	4.4300	2.2419
0.6	1.0000	0.9947	4.8737	0.1554	0.6813	4.2461	2.5155
0.8	0.8617	0.9947	4.8073	0.2129	0.6802	4.1002	2.7138
1	0.7459	0.9947	4.7426	0.2588	0.6817	3.9809	2.8655
2	0.4002	0.9948	4.4761	0.3957	0.6837	3.60735	3.291005
4	0.1381	0.9947	4.1501	0.5034	0.6806	3.2886	3.6055
6	0.0363	0.9947	3.9604	0.5487	0.6767	3.1455	3.7343
8	0.0000	–	3.8372	0.5733	–	3.0640	3.8044
10	0.0000	–	3.7521	0.5887	–	3.0114	3.8485

n	s	f_S	$E(PV(x))$	Premium (percent)	SU(P)	SV(U)
(Subcase C) *Bai*' *Salam* model: α = 2						
0.1	0.0000	–	0.9687	–	−1.0951	−9.3080
0.2	0.0338	0.9996	0.9509	5.1194	−1.1544	−6.1646
0.4	0.1137	0.9771	0.9365	4.3414	−1.2638	−4.1115
0.6	0.1776	0.9698	0.9333	3.9170	−1.3616	−3.3122
0.8	0.2282	0.9686	0.9343	3.6718	−1.4491	−2.8846
1	0.2689	0.9700	0.9370	3.5213	−1.52767	−2.6181
2	0.3883	0.9856	0.9545	3.2618	−1.8241	−2.0612
4	0.4780	1.0094	0.9780	3.2051	−2.1570	−1.7707
6	0.5141	1.0225	0.9908	3.2081	−2.3391	−1.67241
8	0.5335	1.0307	0.9986	3.2146	−2.4539	−1.6230
10	0.5456	1.0361	1.0038	3.2197	−2.5330	−1.5933

Table 2 (*Continued*)

n	s	f_C	r (percent)	Q	Premium (percent)	SU(P)	SV(U)
(Subcase D) Synthetic futures model: α = 2 and $E(x)$ = 0.988							
0.1	1.0000	1.0456	6.4444	−0.0328	5.8330	−1.0837	−8.7077
0.2	1.0000	1.0364	7.5012	0.0144	4.9028	−1.1465	−5.9616
0.4	1.0000	1.0258	7.7699	0.0948	3.8269	−1.2606	−4.0420
0.6	0.9695	1.0213	7.2643	0.1589	3.3656	−1.3610	−3.2779
0.8	0.8467	1.0212	6.6447	0.2116	3.3639	−1.4489	−2.8688
1	0.7490	1.0211	6.0333	0.2540	3.3532	−1.52771	−2.6100
2	0.4818	1.0203	3.5902	0.3816	3.2693	−1.8242	−2.0608
4	0.3261	1.0190	0.8932	0.4841	3.1332	−2.1562	−1.7708
6	0.2835	1.0181	−0.4955	0.5281	3.0514	−2.3373	−1.67235
8	0.2673	1.0176	−1.3308	0.5526	2.9993	−2.4514	−1.6229
10	0.2600	1.0173	−1.8861	0.5681	2.9636	−2.5297	−1.5931

The *Bai' Salam* and synthetic futures models are solved assuming the following exogenous parameters: (1) $w_0 = 2/(1+n)$, $w'_0 = 2n/(1+n)$, $w_1 = 0.4/(1+n)$, $w'_1 = 0.4n/(1+n)$, $\alpha = \alpha_1 = \alpha_2$, d = 1.2 and β = 0.99; (2) the yield and spot prices follow a binomial distribution such that both are negatively correlated as illustrated in Fig. 1. Furthermore, m_1 = 2, m_2 = 1, p_1 = 0.6, q_1 = 0.4, x_1 = 1.1, x_2 = 0.9, p_2 = 0.8, q_2 = 0.2. The endogenous parameters evaluated comprise the following: s (quantity of commodity bid for in either contract), f_S (*Bai' Salam* price), f_C (conventional futures' price), Q (*Bai' Murabahah* funding amount), and r (profit rate). Finally, SU(P) and SV(U) denote the sum of expected utilities of producer and user of commodity, respectively.

Salam contract in the region $\alpha \in [0.1, 1]$ and equally efficient to it in the remaining region of risk aversion, $\alpha \in [2, 5]$.

Case II. Table 2 displays the results where agents are equally risk averse but have unequal wealth. Here, two levels of risk aversion are investigated, namely, α = 0.4 and α = 2. For subcases A and B, where α = 0.4, we cannot solve the system of equations to yield a positive quantity pre-sold (s) for the *Bai' Salam* contract when n = 0.1 and for the futures contract when $n \in [8,10]$. Thus, for n = 0.1, the *Bai' Salam* contract is not feasible, as the Agent 1 is quite wealthy compared to Agent 2. For $n \in [8,10]$, the optimal contract (in case of the synthetic futures model) is a *Bai' Murabahah* one, as the wealthy Agent 2 prefers to advance funds via a credit facility. An interesting situation is observed for $n \in [0.1, 0.2]$, where the wealthy Agent 1 advances funds to agent 2 using the *Bai' Murabahah* facility.

As n increases from 0.1 to 10, f_S depicts a monotonically increasing relationship with respect to s, while f_C shows an inverse relationship with respect to s. The quantity pre-sold in the *Bai' Salam* contract is lower than that in the futures for the region $n \in [0.1, 1]$ and vice versa in the remaining region. The price of the futures contract is higher than that of *Bai' Salam*. However, the premium over the expected present value of spot for *Bai' Salam* is generally higher (with the exception of n = 2). Nonetheless, the synthetic futures model is pareto-optimal over the *Bai' Salam* for the entire region with the exception of n = 2 where it is equally efficient to it.

For subcases C and D where α = 2, we observe that the quantity pre-sold (s) in the synthetic futures model is initially higher but declines below that of the *Bai' Salam* model as n increases to 10. As n increases, f_S is increasing with s while f_C decreases with s. For $n \leq 2$, the quantity bid (s) in the futures model is higher than in the case of *Bai' Salam*, while f_C

is greater than f_S for $n \leq 4$. The situation reverses beyond the values of n stated earlier. The premium of *Bai' Salam*/futures depicts a decreasing trend with n. Here too, conventional futures is pareto-optimal over *Bai' Salam* except for the region $n \in [1,4]$ where it is equally efficient to it.

Table 3
Simulation results (unequal wealth and risk aversion)

n	s	f_S	$E(PV(x))$	Premium (percent)	SU(P)	SV(U)
(Subcase A) *Bai' Salam* model: $\alpha_1 = 0.4$ and $\alpha_2 = 2$						
0.1	0.0000	–	0.9246	–	4.8080	−9.3080
0.2	0.0365	0.9648	0.9243	4.3798	4.6634	−6.1565
0.4	0.1169	0.9571	0.9246	3.5150	4.4285	−4.1117
0.6	0.1814	0.9527	0.9248	3.0127	4.2460	−3.3155
0.8	0.2333	0.9499	0.9249	2.6974	4.0998	−2.8880
1	0.2758	0.9480	0.9250	2.4852	3.9801	−2.6203
2	0.4062	0.9444	0.9257	2.0186	3.6042	−2.0558
4	0.5118	0.9444	0.9278	1.7913	3.2825	−1.7571
6	0.5568	0.9457	0.9296	1.7281	3.1380	−1.6551
8	0.5816	0.9468	0.9310	1.7019	3.0558	−1.6037
10	0.5972	0.9478	0.9320	1.6884	3.0028	−1.5727

n	s	f_C	r (percent)	Q	Premium (percent)	SU(P)	SV(U)
(Subcase B) Synthetic futures model: $\alpha_1 = 0.4$, $\alpha_2 = 2$ and $E(x) = 0.988$							
0.1	1.000	1.0457	4.1786	−0.0341	5.8411	4.8494	−8.6938
0.2	1.000	1.0367	4.6482	0.0128	4.9241	4.6975	−5.9671
0.4	1.000	1.0261	5.1799	0.0934	3.8512	4.4540	−4.0530
0.6	1.000	1.0205	5.4763	0.1573	3.2844	4.2667	−3.2833
0.8	1.000	1.0170	5.6633	0.2083	2.9393	4.1176	−2.8651
1	1.000	1.0148	5.7878	0.2496	2.7095	3.9959	−2.6017
2	1.000	1.0096	6.0071	0.3754	2.1893	3.6151	−2.0429
4	1.000	1.0068	5.9230	0.4775	1.8978	3.2903	−1.7459
6	1.000	1.0057	5.7636	0.5215	1.7955	3.1445	−1.6443
8	1.000	1.0052	5.6276	0.5460	1.7439	3.0617	−1.5930
10	1.000	1.0049	5.5194	0.5616	1.7126	3.0082	−1.5621

n	s	f_S	$E(PV(x))$	Premium (percent)	SU(P)	SV(U)
(Subcase C) *Bai' Salam* model: $\alpha_1 = 2$ and $\alpha_2 = 0.4$						
0.1	0.0000	–	0.9445	–	−1.0951	1.5149
0.2	0.0153	0.9538	0.9352	1.9891	−1.1548	1.8251
0.4	0.1052	0.9512	0.9348	1.7594	−1.2650	2.2343
0.6	0.1734	0.9542	0.9381	1.7141	−1.3629	2.5066
0.8	0.2258	0.9576	0.9414	1.7204	−1.4505	2.7038
1	0.2672	0.9608	0.9443	1.7442	−1.5292	2.8542
2	0.3874	0.9714	0.9534	1.8888	−1.8291	3.2733
4	0.4785	0.9807	0.9609	2.0620	−2.1748	3.5815
6	0.5160	0.9849	0.9643	2.1463	−2.3691	3.7075
8	0.5364	0.9873	0.9661	2.1946	−2.4938	3.7761
10	0.5492	0.9889	0.9674	2.2254	−2.5807	3.8193

Table 3 (*Continued*)

n	s	f_C	r (percent)	Q	Premium (percent)	SU(P)	SV(U)
(Subcase D) Synthetic futures model: α_1 = 2, α_2 = 0.4 and $E(x)$ = 0.988							
0.1	0.5118	1.0102	6.8047	−0.0483	2.2493	−1.0938	1.5227
0.2	0.2191	1.0078	6.2139	0.0109	1.9995	−1.1547	1.8261
0.4	0.0000	–	5.4916	0.1021	–	−1.2649	2.2376
0.6	0.0000	–	5.0057	0.1692	–	−1.3624	2.5120
0.8	0.0000	–	4.5459	0.2212	–	−1.4494	2.7108
1	0.0000	–	4.1534	0.2626	–	−1.5276	2.8623
2	0.0000	–	2.9063	0.3847	–	−1.8241	3.2846
4	0.0000	–	1.8355	0.4791	–	−2.1638	3.5948
6	0.0000	–	1.3459	0.5184	–	−2.3539	3.7215
8	0.0000	–	1.0628	0.5398	–	−2.4756	3.7905
10	0.0000	–	0.8778	0.5533	–	−2.5602	3.8338

The *Bai*' *Salam* and synthetic futures models are solved assuming the following exogenous parameters: (1) $w_0 = 2/(1+n)$, $w'_0 = 2n/(1+n)$, $w_1 = 0.4/(1+n)$, $w'_1 = 0.4n/(1+n)$, d = 1.2 and β = 0.99; (2) the yield and spot prices follow a binomial distribution such that both are negatively correlated as illustrated in Fig. 1. Furthermore, m_1 = 2, m_2 = 1, p_1 = 0.6, q_1 = 0.4, x_1 = 1.1, x_2 = 0.9, p_2 = 0.8, q_2 = 0.2. The endogenous parameters evaluated comprise the following: s (quantity of commodity bid for in either contract), f_S (*Bai*' *Salam* price), f_C (conventional futures' price), Q (*Bai*' *Murabahah* funding amount), and r (profit rate). Finally, SU(P) and SV(U) denote the sum of expected utilities of producer and user of commodity, respectively.

Case III. Table 3 depicts the case where agents have unequal endowments and levels of risk aversion. Cases A and B illustrate the situation when risk aversion of both agents are respectively set at the levels 0.4 and 2, namely, α_1 = 0.4 and α_2 = 2. As n increases from 0.1 to 10, both f_S and f_C decline with increasing/constant 's', respectively. The optimal value of s = 1 in the entire region of n for subcase B illustrates that we have a corner solution where constraint [Eq. (15)] is binding. Here, $f_C > f_S$ and $\text{Premium}_{\text{Futures}} > \text{Premium}_{Bai'\,Salam}$. Nonetheless, conventional futures is pareto-optimal over *Bai*' *Salam*.

Subcases C and D illustrate the situation where the level of risk aversion is reversed from the previous subcases to 2 and 0.4, respectively, namely α_1 = 2 and α_2 = 0.4. Here again, the market clearing constraint $s \geq 0$ [Eq. (15)] is binding, and we observe a lack of equilibrium for *Bai*' *Salam* for n = 0.1 and feasibility of *Bai*' *Murabahah* only for the region $n \in$ [0.4, 10]. Here too, the synthetic futures model is pareto-optimal over that of the *Bai*' *Salam* model.

Case IV. Finally, sensitivity analysis of the base case (Table 1) is done by perturbing the various exogenous parameters such as m_1, m_2, d, x_1, x_2, w_0, w_1, p_1 and p_2. The results (not reported) indicate that changes in endogenous parameters noted are not drastically different from the solutions illustrated in the above tables.

5.3.1. *Further explication of the above results*

Two key results emerge from the above simulation. They are that (i) both *Bai*' *Salam* and futures are priced at a premium to the expected spot; and (ii) *Bai*' *Salam* is pareto-inferior to synthetic futures. These need to be, respectively, elucidated below from an economic framework:

(i) Our assumption of negative correlation between the joint quantity/yield risk and price risk (stemming from an ‘inferior’ type of commodity) is the source of the premium. This is because the negative correlation between quantity and price partially reduces the risk exposure of total revenue of the producer as well as the user. In this case, *Bai*’ *Salam* or futures contracting increases risk exposure of total revenue of both producer as well as user. The only way to accomplish this is to compensate the parties involved with a premium over expected spot. In contrast, if we had a case of positive correlation between quantity risk and price risk (i.e., the case of ‘normal’ commodity), then we would have observed a discount to the expected spot as both producers and users would prefer to contract at a discount to reduce their overall risk exposure.

(ii) The *Bai*’ *Salam* and futures packages are priced by using their necessary conditions given in Propositions I and II, namely, Eqs. (17)–(19) respectively. This yields f_S, f_C, Q and r as non-linear functions of coefficient of risk aversion and wealth of agents. Since a synthetic futures package is comprised of a portfolio of credit and futures facilities, it can be construed as having more flexibility (or less restrictive) than a *Bai*’ *Salam* facility. Thus, the optimization of a conventional futures package is akin to an unconstrained one in contrast to *Bai*’ *Salam* whose optimization is akin to a constrained one. This is alluded to in our Theorem and is the precise reason why the conventional futures package is pareto-optimal to that of the *Bai*’ *Salam* security.

6. Concluding remarks

This study investigates the efficiency of conventional futures over that of the classical *Bai*’ *Salam* contract. Despite the subtle difference in the timing of the payment of the two contracts, we find the futures contract to be pareto-optimal. This is due to the fact that the futures contract is more flexible under agent heterogeneity in the form of level endowments (wealth) and risk aversion. This result has implications for mainstream economists as it implies that the concept of arbitrage needs to be re-examined under non-linear asset pricing. Furthermore, our result, identifying futures as a quasi-equity claim, eliminates a major hurdle against their implementation: their classification as debt (*dayn*) by the majority of the contemporary Islamic scholars. Our results are consistent with the framework of the Islamic law (*Shari*’*ah)* as elaborated earlier. Since the objective (*maqsad*) of the Islamic law (*Shari*’*ah*) (as advocated by Ibn Qayyim Al-Jawziyya) is the welfare of the people in this world as well as in the hereafter, we conclude that the welfare of the emerging Muslim economies would be reinforced by substituting modern futures on Islamically allowed commodities for *Bai*’ *Salam*. Currently four Muslim countries (Indonesia, Kazakhstan, Malaysia and Turkey) have initiated the same in a limited way (Bacha, 2002; Kamali, 2002; Peck, 2000). The remaining Muslim countries need to follow their trend to benefit from the effects of financial deepening. The welfare gains anticipated by modernizing the financial intermediation of emerging Muslim countries would lead to their economic expansion.

Our results also have major implication for global banking industry in general and Islamic banking in particular. In the last two decades, Islamic banks have grown in size and number around the world. Even conventional commercial banks from developed countries have started to offer Islamic banking services. Islamic banking and its expansion world-wide depend on the ability of the industry to develop and offer a wide range of more creative and competitive products. The introduction of a financially engineered package consisting of conventional futures contracts on Islamically permissible commodities and an Islamic credit facility discussed in this paper will allow the industry to move in the right direction.

Acknowledgments

We are extremely grateful to J. Barkley Rosser Jr. (the Editor) and an anonymous referee for helping us in improving this paper significantly. We also appreciate the research material and critical comments (on earlier drafts of the paper) received from Muhammad Anwar, Obiyathulla Bacha, Masudul Alam Choudhury, Mahmoud El-Gamal, Sudin Haron, Zamir Iqbal, Tariqullah Khan, Sami Al-Suwailan, Usamah Uthman and Rodney Wilson.

Appendix A. Glossary of Islamic finance terms

Islamic finance terms	Interpretation
Al-kharaj bi al-daman	Entitlement of return from an asset vests on one bearing risk of it
Bai'	Sale
Bai' al dayn bi al dayn	Sale of one form of debt with another or debt securitization
Bai' Murabahah	Cost-plus sale
Bai' Salam	Islamic forward sale
Dayn	Debt
Gharar	Deception
Hadith/Sunnah	Sayings, deeds or tacit approvals of Prophet Muhammad (PBUH)
Hasan	Good
Ibahah	Permissible
Ijtihad	Exertion of efforts by a qualified scholar to deduce the law that is not self evident from its sources
Istihsan	To deem something good or equitable in Islamic law
Maqsad	Objective
Maysir/qimar	Gambling
Musharakah	Equity (ownership claim) in a business or venture
Qabd	Possession

Appendix A (*Continued*)

Islamic finance terms	Interpretation
Qur’an	Muslim holy book
Riba-al-fadl	Involves unfair trade, market manipulation or trade under duress
Riba-an-nasi’ah or Ribawi	Financial claims based on fixed and pre-determined return such as conventional interest rates
Shari’ah	Islamic law

References

Aggarwal, R.K., Yousef, T., 2000. Islamic banking and investment financing. Journal of Money, Credit and Banking 32, 93–120.

Al-Alwani, T.J., 1997. The role of *Ijtihad* in the regulation and correction of capital markets. American Journal of Islamic Social Sciences 14, 39–66.

Anon., 1901. Jewish Encyclopedia, vol. 12. Ktav Publishing House Inc., New York.

Bacha, O.I., 1999. Derivative instruments and Islamic finance: some thought for a reconsideration. International Journal of Islamic Financial Services 1, 9–25.

Bacha, O.I., 2002. Financial Derivatives: Markets and Applications in Malaysia. Penerbit Universiti Putra Malaysia, Selangor Darul Ehsan, Malaysia (pp. 12–24).

Breeden, D.T., 1980. Consumption risk in futures markets. Journal of Finance 35, 503–520.

Chiang, A.C., 1984. Fundamental Methods of Mathematical Economics. McGraw-Hill, New York.

Demetriades, P., Arestis, P., Fattouh, B., 2000. A good financial system can contribute at least 0.5% a year to economic growth. In: Paper presented at the Annual Meeting of the Royal Economic Society. St. Andrews, Scotland, UK.

El-Gari, M.A., 1997. Short-term financial instruments based on Salam contracts. In: Khan, T., Ahmad, A. (Eds.), Islamic Financial Instruments for Public Sector Resource Mobilization. Islamic Research and Training Institute, Islamic Development Bank, Jeddah, Saudi Arabia, pp. 249–266.

Errico, L., Farahbaksh, M., 1998. Islamic banking: issues in prudential regulations and supervision. In: Proceedings of the Second Harvard University Forum on Islamic Finance. (pp. 183–194).

Fazlur-Rahman, 1969. *Riba* and interest. Islamic Studies 3, 1–43.

Francis, D., 2000. The gains from futures trading. In: Goss, B.A. (Ed.), Models of Futures Market. Routledge, London, UK, pp. 15–41.

Goss, B.A., 2000. Introduction: welfare, rationality and integrity in futures markets. In: Goss, B.A. (Ed.), Models of Futures Market. Routledge, London, UK, pp. 1–14.

Hastings, J., 1922. Encyclopedia of Religion and Ethics, vol. 12. Scribner’s Sons, New York.

Hirshleifer, J., 1975. Speculation and equilibrium: information, risk and markets. Quarterly Journal of Economics 89, 519–542.

Hirshleifer, D., 1988. Residual risk, trading costs, and commodity futures risk premia. Review of Financial Studies 1, 173–193.

Ibn Qayyim Al-Jawziyya, 1973. *I’lā m Al-Muwaqqa’ī n ’alā Rabb al-’ā lam’ī n*. In: *Taha ’Abd Al-Ra’uf Sa’d* (Ed.), Dar Al-Jil, Beirut, Lebanon (vol. II, pp. 153–164, vol. III, p. 4).

Ibn Taymiya, 1951. *Al-Qawā ’id Al-Nū rā niyya Al-Fiqhiyya*. In: al-Fiqi, M. (Ed.), *Al-Sunna Al-Muhammadiyya*. Cairo, Egypt, pp. 112–133.

Ibn Taymiya, n.d. *Nazariyyat Al-’Aqd*. Dar Al-Ma’rifa, Beirut, Lebanon, pp. 220–229.

Islahi, A.A., 1988. Economic Concepts of Ibn Taimiyah. Islamic Foundation, Leicester, UK (pp. 123–145).

Kamali, M.H., 1996. Islamic commercial law: an analysis of futures. American Journal of Islamic Social Sciences 13, 197–224.

Kamali, M.H., 2000. Principles of Islamic Jurisprudence. Ilmiah Publishers, Selangor Darul Ehsan, Malaysia (p. 250).

Kamali, M.H., 2002. Islamic Commercial Law: An Analysis of Futures and Options. Ilmiah Publishers, Selangor Darul Ehsan, Malaysia.

Keen, S., 1997. From prohibition to depression: the Western attitude to usury. Accounting, Commerce and Finance: The Islamic Perspective 1, 26–55.

Khan, M.F., 1997. Islamic futures markets as means for mobilizing resources for development. In: Khan, T., Ahmad, A. (Eds.), Islamic Financial Instruments for public sector resource mobilization. Islamic Research and Training Institute, Islamic Development Bank, Jeddah, Saudi Arabia, pp. 133–168.

Kocherlakota, N.R., 1996. The equity premium: it's still a puzzle. Journal of Economic Literature 34, 42–71.

Kolb, R.W., 1996. The systematic risk of futures contracts. Journal of Futures Markets 16, 631–654.

Lucas Jr., R.E., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

Mehra, R., Prescott, E.C., 1985. The equity premium: a puzzle. Journal of Monetary Economics 15, 145–161.

Peck, A.E., 2000. The development of commodity futures exchanges in Kazakhstan and China. In: Goss, B.A. (Ed.), Models of Futures Market. Routledge, London, UK, pp. 42–60.

Ritchken, P., 1996. Derivative Markets: Theory, Strategy and Applications. Harper Collins, New York (p. 38).

Saeed, A., 1996. Islamic Banking and Interest: A Study of the Prohibition of *Riba* and its Contemporary Interpretation. E.J. Brill, Leiden, The Netherlands (pp. 17–50, 76–95, 119–141).

Siddiqi, M.N., 2001. Towards a grass-roots based Islamic finance for all. In: Eighth Annual International Conference of *Lariba* Islamic Banking (Keynote address). Los Angeles, CA.

Siddiqi, M.N., 2002. Comparative advantages of Islamic banking and finance. In: Paper presented at the Fifth Harvard University Forum on Islamic Finance. Cambridge, MA.

Siddiqui, A.H., 1986. Sahih Muslim-Translation. Kitab Bhavan, New Delhi, India (vol. III, pp. 844–845 (*Hadith* # 3906), vol. IV, pp. 1259–1260 (*Hadith* # 5831)).

Siegel, D.R., Siegel, D.F., 1994. The Futures Market: The Professional Trader's Guide to Portfolio Strategies, Risk Management and Arbitrage. Probus Publishing Company, Chicago, IL (pp. 127–130).

Udovitch, A.L., 1975. Commercial techniques in early medieval Islamic trade. In: Richards, D.S. (Ed.), Islam and the Trade of Asia: A Colloquium. Oxford, UK, pp. 37–62.

Varian, H.R., 1987. The arbitrage principle in financial economics. Journal of Economic Perspectives 1, 55–72.

Vogel, F.E., Hayes III, S.L., 1998. Islamic Law and Finance: Religion, Risk and Return. Kluwer Law International, Hague, The Netherlands (p. 171).

Whalley, J.J., 1988. Lessons from general equilibrium models. In: Aaron, H., Galper, H., Pechman, J. (Eds.), Uneasy Compromise. The Brookings Institution, Washington, DC, pp. 15–50.

Yasseri, A., 2000. Islamic banking contracts as enforced in Iran. In: Iqbal, M., Llewellyn, D.T. (Eds.), Islamic Banking and Finance: New Perspectives on Profit Sharing and Risk. Edward Elgar, Northampton, MA, pp. 155–168.

Zaman, S.M.H., 1991. *Bay' Salam*: principles and practical application. Islamic Studies 30, 443–461.